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# THE SIMPLEST MODELS AND SIMPLIFIED METHODS FOR DETERMINING THE NONLINEAR SHEAR CREEP OF CLAY SOILS

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The Drucker–Prager diagram of an ideal elastoplastic body [1] is used to describe the dependence of a shear stress $\tau$ on a clay-soil shear strain $\gamma$ with allowance for the variability of a density–humidity state under the action of normal stresses $\sigma_z$. In the diagram of [1], the effect of $\sigma_z$ on the dependence $\gamma - \tau$ is expressed in terms of the change in the ultimate shear stress $\tau_f$ (yield point), while in the diagram of S. S. Vyalov [2], it is expressed in terms of the change of both the elasticity modulus (linear deformation) $G$ and the quantity $\tau_f$.

Real clay soils are characterized by the $\sigma_z$-dependent shear-creep property $\gamma_t = f(t)$ and the nonlinear dependence $\gamma_t - \tau$. For $\sigma_z = \text{const}$, the $\gamma_t - \tau$ dependence is determined from one family of experimental shear-creep curves (Fig. 1) at various fixed times. The effect of $\sigma_z$ on both $\gamma_t - \tau$ and $\gamma_t - t$ can be taken into account using several families of experimental shear-creep curves which are constructed at various constant normal stresses ($\sigma_{z,i} = \text{const}$) [3]. In the present paper, we consider the two simplest models and simplified experimental methods of determining the characteristics of the nonlinear shear creep of clay soils taking into account the variability of their states under the action of normal stresses $\sigma_z$.

According to the theory of hereditary creep of clay soils whose properties vary under the action of $\sigma_z$, the expression for the shear strain variation in time $\delta(t - \vartheta, \sigma_z)$ under the action of a unit shear stress $\tau = 1$ (similarly to the strain of aging materials in [4]) has the form [3, 5]

$$\delta(t - \vartheta, \sigma_z) = 1/G_0(\sigma_z) + \omega(t - \vartheta, \sigma_z) = 1/G_0(\sigma_z) + \varphi(\sigma_z)\psi(t - \vartheta), \qquad (1)$$

where $1/G_0(\sigma_z)$ is the $\sigma_z$-dependent instantaneous shear-strain, $\omega(t - \vartheta, \sigma_z)$ is the $\sigma_z$-dependent shear creep measure ($\tau = 1$), $\varphi(\sigma_z)$ is a state function, $\psi(t - \vartheta)$ is a function of time; $t$ is the time, and $\vartheta$ is the loading time.

If the instantaneous (elastic) strains are ignored because of their smallness [2], the expression for shear creep (with $\vartheta = 0$) has the form [3, 5]

$$\gamma_t(t, \tau, \sigma_z) = \omega(t, \sigma_z) f(\tau, \sigma_z) = \varphi(\sigma_z)\psi(t) f(\tau, \sigma_z). \qquad (2)$$

In this case, $f(\tau, \sigma_z)$ is the $\sigma_z$-dependent function of shear stress which takes into account the $\gamma_t - \tau$ nonlinear dependence and is subject to the condition $f(\tau = 1, \sigma_z) = 1$.

Expression (2), which is represented in the simplest form of writing of the aging theory, establishes a relationship between the shear stress, the nonlinear shear creep, time, and the normal stress. It can be successfully used to determine shear strains under the action of both constant and slowly increasing shear stresses.

The $\gamma_t - \tau$ dependence (Fig. 1a) is written, in particular, as a power function

$$\gamma_t(\tau) = B\tau^n. \qquad (3)$$

For $f(\tau)$, we then obtain

$$f(\tau) = \gamma_t(\tau)/[\gamma_t(\tau = 1)] = \tau^n. \qquad (4)$$

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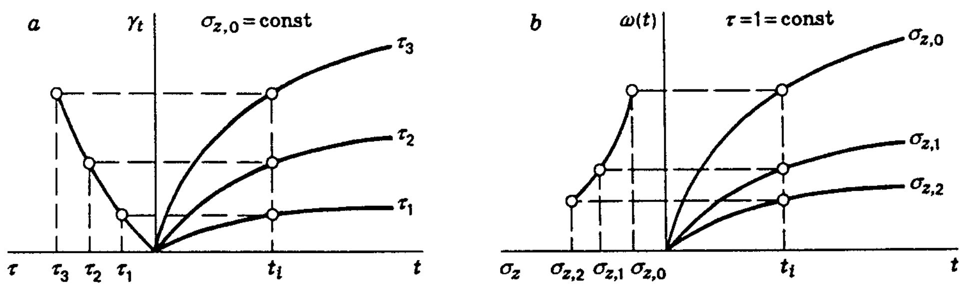


Fig. 1

As early as 1960, the author elucidated that for clay soils, the shear-creep nonlinear-strain index $n$ does not actually depend on their state (of $\sigma_z$) [3]. This was later confirmed by many experiments. In particular, in experiments with Kiev clay ($\rho_s = 2670$ kg/m$^3$, $\rho = 1920$–$1980$ kg/m$^3$, $w = 0.25$–$0.32$, $w_L = 0.537$, $w_P = 0.333$, and $I_P = 0.204$) and with Novomikhailovskii argillaceous clay ($\rho_s = 2610$ kg/m$^3$, $\rho = 1640$–$1740$ kg/m$^3$, and $w = 0.329$) [6], the values of $n$ (Table 1) were obtained for various $\sigma_z$ by means of M-5 torsion devices [5].

Thus, the nondependence of the shear-stress function (4) on the state of the soil ($\sigma_z$) was revealed. This allows one to conclude that it suffices to have one experimental family of shear-creep curves to determine $f(\tau)$.

Using such a family (Fig. 1a), we determine the $\gamma_t - \tau$ dependence, the shear-stress function $f(\tau)$, and the shear-creep measure $\omega(t, \sigma_z = \text{const})$ for a given state of soil. To find the dependence of the shear-creep measure on $\sigma_z$, we need a family of experimental curves of the shear-creep measure (Fig. 1b), from which the $\omega(t) - \sigma_z$ and $\varphi(\sigma_z)$ dependences are established.

If, in particular, the $\omega(t) - \sigma_z$ dependence is represented as

$$\omega(t, \sigma_z) = (t, \sigma_{z,0}) - C(\sigma_z - \sigma_{z,0})^{n_1}, \tag{5}$$

for $\varphi(\sigma_z)$, we obtain the equation

$$\varphi(\sigma_z) = 1 - C_1(\sigma_z - \sigma_{z,0})^{n_1}, \tag{6}$$

subject to the condition $\varphi(\sigma_z) = 1$ for $\sigma_z = \sigma_{z,0}$, where $\sigma_{z,0}$ is the minimal normal stress at which the initial expression for the shear-creep measure $\omega(t, \sigma_z = \sigma_{z,0})$ was derived.

From the foregoing it follows that it is sufficient to perform a creep test in one state of soil ($\sigma_{z,0}$ = const) for not less than three twin specimens under various shear stresses to construct a family of shear-creep curves (Fig. 1a) and to determine expression (2). To obtain a family of shear-creep-measure curves (Fig. 1b), it is necessary to test for creep not less than two additional specimens at $\tau = 1$ exposed to the stresses $\sigma_{z,i=1,2...}$ which differ from the stresses $\sigma_{z,0}$. The number of specimens can be reduced if one uses the methods of one or two experimental curves to find a family of shear-creep curves.

The right-hand side of Figs. 2–4 shows, as an example, the deduction of expression (2) using the method of two experimental curves [5]. The solid shear-creep measure curves with circles were experimentally obtained, respectively, in three states ($\sigma_z = 0.15$, 0.25, and 0.35 MPa) of the 46-75 clay ($\rho_s = 2810$ kg/m$^3$; $\rho = 1850$ kg/m$^3$; $w = 0.41$, $w_L = 0.555$, $w_P = 0.374$, $I_P = 0.181$, $\varphi = 15^\circ 40'$, and $c = 0.012$ MPa) under the action of constant and stepwise increasing shear stresses. The left-hand side of Figs. 2–4 shows the $\gamma_t - \tau$ curves approximated by expression (3). The shear-stress function

$$f(\tau) = B_1(10\tau)^n \tag{7}$$

that is subject to the condition $f(\tau = 0.5\,\text{MPa}) = 1$ is defined. In (7), $\tau = 0.05$ MPa is regarded as a unit shear stress.

Table 2 summarizes the experimental data on the standard resistance to the shear $\tau_{f,st}$ along with the parameters $B$, $B_1$, and $n$ which were obtained by testing of solid specimens of diameter 101 mm and height

TABLE 1

| Soil | $\sigma_z$, MPa | | | | | | Average values |
|---|---|---|---|---|---|---|---|
| | 0.30 | 0.32 | 0.5 | 0.55 | 0.8 | 1.2 | |
| | $n$ | | | | | | $n$ |
| Kiev clay | — | 2.995 | — | 2.866 | 2.931 | 2.931 | 2.931 |
| Novomikhailovskii argillaceous soil | 2.209 | — | 2.00 | — | 2.285 | — | 2.17 |

TABLE 2

| $\sigma_z$, MPa | $\tau_f$, MPa | $B$ | $B_1$ | $n$ | $a$ | $b$ |
|---|---|---|---|---|---|---|
| 0.15 | 0.055 | 1.687 | 27.88 | 4.80 | 0.0045 | 0.0040 |
| 0.25 | 0.083 | 2.31 | 27.27 | 4.77 | 0.0040 | 0.0040 |
| 0.35 | 0.111 | 0.0609 | 25.80 | 4.69 | 0.0056 | 0.0049 |

24 mm using M-5 torsion devices.

The dashed shear-creep curves were constructed at constant normal stresses $\sigma_z = 0.15$, 0.25, and 0.35 MPa under the action of the constant shear stresses $\tau = 0.034$, 0.0515, and 0.0688 MPa (Figs. 2–4) and were approximated by the logarithmic dependences

$$\gamma_t(t) = a + b \log t. \tag{8}$$

The parameters $a$ and $b$ of expression (8) are also given in Table 2.

Using (7) and (8), from (2) we derive expressions for the shear-creep measure for three states of the 46-75 soil:

$$\omega(t, \sigma_{z,0} = 0.15 \text{ MPa}) = 0.0286 + 0.0254 \log t; \tag{9a}$$

$$\omega(t, \sigma_{z,1} = 0.25 \text{ MPa}) = 0.0035 + 0.0035 \log t; \tag{9b}$$

$$\omega(t, \sigma_{z,2} = 0.35 \text{ MPa}) = 0.00125 + 0.00108 \log t. \tag{9c}$$

Expressions (9) were obtained from (2), because the specimens that are characterized by different states of soil were not tested at the same value of the constant shear stress. To avoid this, the specimens should be tested in all states of the soil under the same unit shear stress.

Using relations (7) and (9) along with the expression $\gamma_t = \omega(t, \sigma_z = \text{const}) f(\tau)$, we plotted (the dashed curves on the right-hand side of Figs. 2–4) the shear-creep curves for various constant shear stresses. The solid curves on the right-hand side of Fig. 5 refer to the shear-creep measure and were calculated by (9), and the left-hand side shows the $\omega(t) - \sigma_z$ curve. This curve was approximated by relation (5) to derive the expression for a state function of type (6)

$$\varphi(\sigma_z) = 1 - 1.194(\sigma_z - 0.15)^{0.137}, \tag{10}$$

subject to the condition $\varphi(\sigma_z = 0.15 \text{ MPa}) = 1$.

The dashed curves in Fig. 5 show the shear-creep measure ($\tau = 0.05$ MPa) for $\sigma_{z,1} = 0.25$ and $\sigma_{z,2} = 0.35$ MPa and were constructed using the expression $\omega(t, \sigma_z) = \omega(t, \sigma_{z,0})\varphi(\sigma_z)$ with allowance for (9a) and (10).

Using expressions $f(\tau, \sigma_{z,0} = 0.15 \text{ MPa})$ (7) and $\omega(t, \sigma_{z,0} = 0.15 \text{ MPa})$ (9a) for one state of the soil (see Fig. 2) and also the state function (10) defined from the family of creep-measure curves, we write relation (2) as

$$\begin{aligned} \gamma_t(t, \tau, \sigma_z) &= \varphi(\sigma_z)\omega(t, \sigma_z = 0.15 \text{ MPa}) f(\tau, \sigma_z = 0.15 \text{ MPa}) \\ &= [1 - 1.194(\sigma_z - 0.15)^{0.137}](0.0286 + 0.0254 \log t) 27.88 (10\tau)^{4.8}. \end{aligned} \tag{11}$$

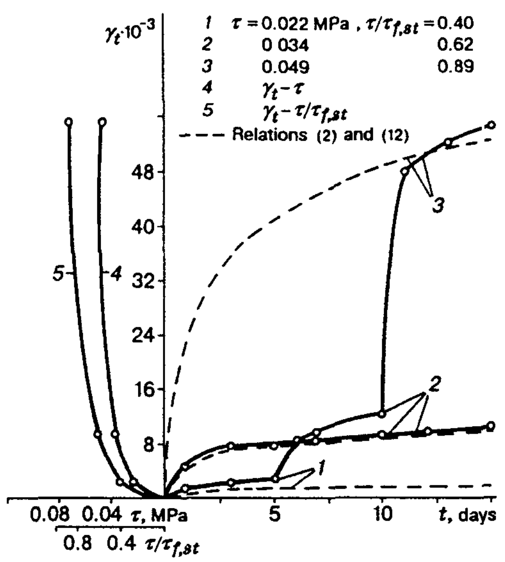


Fig. 2

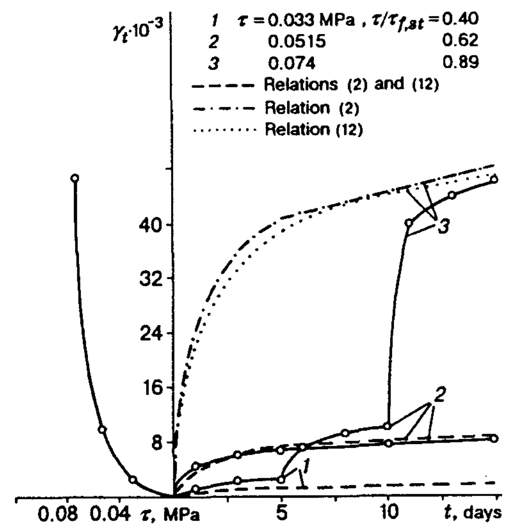


Fig. 3

The expressions for the shear creep were derived from relation (11) for $\sigma_z = 0.25$ and 0.35 MPa for various values of the constant shear stress. The curves plotted using these expressions practically coincide with those constructed using the method of two experimental curves for these states of soil (the dashed curves in Figs. 3 and 4).

It has been verified experimentally [5–8] that apart from the nondependence of the shear-stress function $f(\tau)$ on the clay-soil state $\sigma_z$, its shear-creep strains, determined at the same levels of shear stress $\tau/\tau_{f,st}$ ($\tau_{f,st}$ is the standard shear resistance), are also independent of $\sigma_z$. Thus, a generalized equation for the shear creep has the form [5, 6]

$$\gamma_t = \omega(t) f(\tau/\tau_{f,st}), \tag{12}$$

where $f(\tau/\tau_{f,st})$ is a shear-stress function, subject to the condition $f(\tau/\tau_{f,st} = 1) = 1$, $\omega(t)$ is the measure of the unit level of shear creep, and

$$\tau_{f,st} = \sigma_z \tan\varphi + c. \tag{13}$$

The shear-creep measure for $\tau/\tau_{f,st} = 1$ is fictitious and can be used only to determine strains at $\tau/\tau_{f,st} < 1$. Expression (12), which is represented, as (2), in the form of writing of the aging theory, relates the shear stresses, the nonlinear shear strains, time, and the soil resistance to shear, and we employ it to take into account the variability of the state of soil under the action of $\sigma_z$.

If the $\gamma_t - \tau/\tau_{f,st}$ dependence is expressed by means of (3), the function of the level of shear stress with allowance for (13) takes the form

$$f\left(\frac{\tau}{\tau_{f,st}}\right) = \left(\frac{\tau}{\tau_{f,st}}\right)^n = \left(\frac{\tau}{\sigma_z \tan\varphi + c}\right)^n. \tag{14}$$

Obviously, to derive (12), it is sufficient to have both only one family of experimental shear-creep curves (Fig. 1) obtained by testing of specimens for various levels of shear stress and the diagram of the soil resistance to shear which is needed to find the parameters $\varphi$ and $c$.

To illustrate the derivation of (12), we shall make use of the family of shear-creep curves of the 46-75 soil which were plotted for $\sigma_{z,0} = 0.15$ MPa (see Fig. 2). In this case, one experimental curve was constructed at a constant level of shear stress $\tau/\tau_{f,st} = 0.62$, and the other was plotted with a stepwise increasing level: $\tau/\tau_{f,st} = 0.4$, 0.62, and 0.89. The left-hand side of Fig. 2 shows the $\gamma_t - \tau/\tau_{f,st}$ curve. For the function

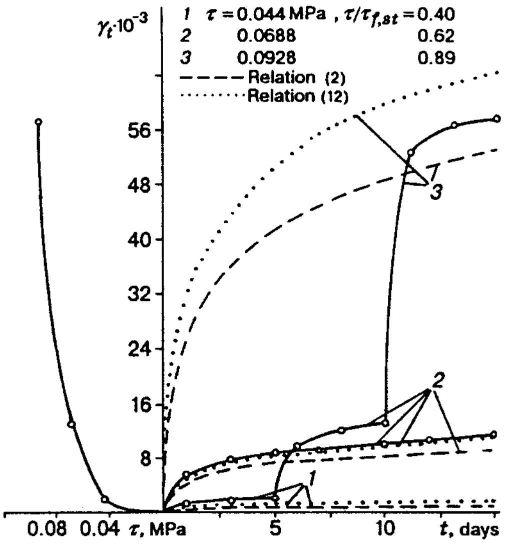


Fig. 4

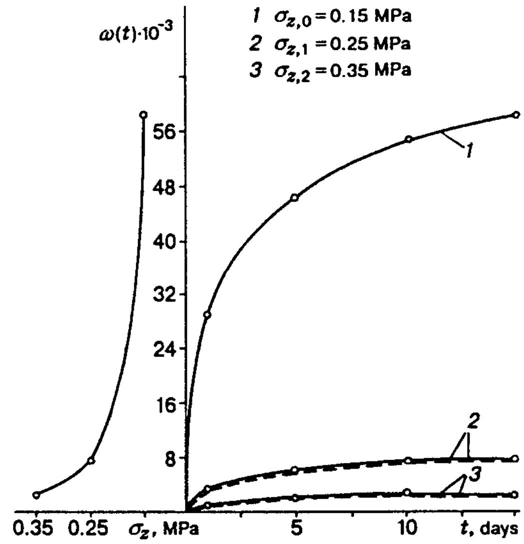


Fig. 5

$f(\tau/\tau_{f,st})$, we derived an expression of the type (14):

$$f(\tau/\tau_{f,st}) = (\tau/\tau_{f,st})^{4.8}. \tag{15}$$

Approximating the shear curve for $\tau/\tau_{f,st} = 0.62$ by (8), whose parameters are listed in Table 2, and using the function (15), we obtain the expression for the level of shear-stress measure

$$\omega(t, \tau/\tau_{f,st} = 1) = 0.045 + 0.04 \log t. \tag{16}$$

The shear-creep curves, constructed using relation (12) and with allowance for (15) and (16), for all three states of the 46-75 soil with the levels of shear stress $\tau/\tau_{f,st} = 0.4$, 0.62, and 0.89 are shown by the dotted curves or coincide with the dashed curves in Figs. 2–4.

We finally conclude that the simplest models of nonlinear clay-soil shear creep (2) and (12) are not practically different from each other in both their complexity and the results of approximation of the experimental data. At the same time, relation (12) has some advantage over relation (2): the number of shear creep experiments is minimal, and the determination of the shear-resistance parameters $\varphi$ and $c$ is of great importance for laboratory use.

The proposed models are noted for a clear approach to solution of the problem, simple forms and methods for determining a small number of parameters, and also for an exact approximation of experimental data. Application of the simplified methods of one and two experimental curves to estimation of the shear creep of clay soils substantially reduces the scope of work and makes them applicable to laboratory practice.

Note that taking into account the shear-stress variation with time complicates the models [4, 5], and the obtained results can be extended to the case of a complex stress-strain state of soils.

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